**Reviewer Comments**

**Reviewer: 1**

Recommendation: Author Should Prepare A Minor Revision

Comments:

To the best of my knowledge the decomposition of shell models to achieve support-free additive manufacturing is a relevant problem that was never taken into account before. Nice catch.

For the algorithmic part. From my experience mean curvature skeletons tend to be extremely smooth and to miss lots of details. This may prevent the decomposition system to find a legal solution (a sub-graph that isolates such features does not exist in the search space). This is mentioned in the limitations, although I would argue that the depicted model is not the right example. I am truly convinced that the laplacian skeleton would catch all those tiny spykes, whereas bigger and smoother protuberances (possibly exceeding the overhang threshold) may not be caught and ask for a considerable amount of supports.

That being said, the authors show a variety of results, most of which look nice (despite some nasty artifacts around seams in Figure 12). I just felt a little disappointed when I realized that no lightweight mechanical component appears in the results. These components play a central role in the motivational part of the paper, so I would have expected to see some fabricated example sooner or later. Why this is not the case?

Overall, the paper is well written and shows nice results. I would recommend it for publication, subject to minor revision. Specifically, the authors should:

- include an example of mechanical lightweight component

- propose a deeper analysis on the limitations, especially regarding the reliability of the skeleton-based method

We have added a shell mechanical component, i.e., a bearing seat, as a new example in the revision. A deeper analysis is provided in the conclusion.

Minor comments:

- The proof that the problem is NP-Hard is a little bit in the middle of things. I think it is interesting to show it, but I would move it to a separate section or to the appendix.

We have carefully considered this suggestion, but we feel that the proof constitutes one of the contributions and the current placement of the proof is all right for the format of the paper.

- The number of skeleton arcs reported in Table 2 seems pretty high. Does it correspond to the number of skeleton bones (i.e., a Y shape would have 3 bones), or rather to the number of tiny little segments each piece-wise linear skeleton is composed of?

It is the latter.

- Perhaps this reference on shape decomposition and re-assembly for 3D printing is missing:

Shapes in a box: disassembling 3D objects for efficient packing and fabrication

Computer Graphics Forum 34 (8), 64-76, 2015

We have added the reference in the fourth section of “Related Work”.

- page 1 line 10: “…and time, partition-based…” => “…and time. Partition-based…”

- page 1 line 13: industrial => industry

- page 1 line 19: optimistic => optimal

- page 1 line 29: stereolithographic => stereolithography

- page 1 line 45: literatures => methods?

- page 2 line 46: “reducing a known NPC problem to it” => “reducing it to a known NP-Complete problem”

- page 3 line 22: an cylindrical => a cylindrical

- page 8 line 6: counts => count

- page 8 line 30: our humans => humans

All are revised.

**Reviewer: 2**

Recommendation: Author Should Prepare A Major Revision For A Second Review

Comments:

Detailed comments:

Sec. 1

"Although 3D printing has seen its applications in producing arbitrarily intricate 3D models, the price of the printing materials, especially for those with high quality, are still outrageously high. " As not all readers would be very familiar with 3D printing, it would be good to qualify what 'outrageously high' means in some more quantitative way.

We have given the price ranges of the major types of materials in the first paragraph of the section for “Introduction”.

Concerning the related work (in this section and the next one), a very relevant paper which should be discussed is:

Voxel-Based Assessment of Printability of 3D Shapes

(A. Telea, A. Jalba)

International Symposium on Mathematical Morphology and Its Applications to Signal and Image Processing (ISMM 2011), Springer, pp 393-404

The respective work is related to quantifying the volume of the printed shape in a local fashion (that is, assessing the so-called wall thickness). This is (incidentally) also related to the use of skeleton structures, which are locally centered in the shape. The current paper could make a (justified) claim that their use of skeletons is a more accurate/detailed way to analyze a 3D shape than the relatively simpler distance-transform-based techniques used in the above ISMM publication.

We have added the relevant references and discussions among the suitability of various skeletons (Section 3).

The introduction should also formally define what a shell model is (even for the fact that not all readers are supposed to directly know what that is).

We have added more explanation on this. Refer to the third paragraph of the section for “Introduction”.

Fig. 1 and related text: from the current text/context, I cannot really see (a) what is precisely the 'roof' of the shape in Fig. 1b (I can guess that's the horizontal/slanted part atop of the shape, but I'm not 100% sure) and (b) why the roof precisely needs support (again, I can guess why, but this should not be left to one's interpretation/imagination).

We have revised it into “overhang region”.

**"3D boundary-represented mesh" isn't a mesh always and per definition a boundary representation of something?**

We have revised it into shell model.

More importantly, the requirement of 'being free from support materials' should be very clearly explained, possible by a figure. Or, alternatively more explanations can be added to Fig. 2 to accomplish this.

We have added more explanation for it the first time we mentioned it. Refer to the third paragraph in the section for “Introduction”. We use the term “support-free” by convention in the 3D printing area.

"...and any searching algorithm could quickly lead to expediential space. " It's not clear to me why this would be the case, since we haven't seen so far what kind of search is needed, and why. Also, what is 'expediential'? (exponential maybe?)

We have revised it. It should be “exponential time”. Refer to the fifth paragraph of the section for “Introduction”.

"We restrict our focus to articulated or organic models whose shapes merit well-defined skeletons" I know precisely (I believe) what the meaning of this is, but as formulated, it's not correct. First, any 3D closed shape does admit a skeleton, and actually there are multiple types of skeletons admitted by such shapes (e.g. surface skeletons, curve skeletons). Please see e.g.

3D Skeletons: A State‐of‐the‐Art Report

A Tagliasacchi, T Delame, M Spagnuolo, N Amenta, A Telea

Computer Graphics Forum 35 (2), 573-597

...where all these are discussed. I believe that the authors mean curve skeletons here. But even so, any 3D shape admits a curve skeleton (see above survey). What I think the meaning is, is to say that, for locally non-tubular shapes, the curve skeleton doesn't describe the shape's geometry well enough to allow meaningful analyses and processing thereof.

Yes, that’s right. Refer to paragraph 5-6 in the section for “Introduction”, we have added more explanation on this. More discussion on the use of various types of skeletons is provided in the section for “Choice of skeletons” (Section 3). We have also added this survey into the reference.

"compute a minimum set of subgraph of the skeleton" Please clarify, it's unclear what is this minimum set. Also, I don't quite see how Fig. 4 helps explaining this sentence.. something is missing in the explanation or caption of Fig. 4.

It is the minimum number of subgraphs. Refer to paragraph 7 in the section for “Introduction”. We have added more explanation on this.

Sec. 2

I found the related work to be clear and to the point. The only (very) related paper that I know of, and which should be added for completeness of exposition of recent results, is the one on 3D printability/wall thickness, which is mentioned earlier in this review.

We have added reference [1] to the “Related Work” section in the revision.

Sec. 3

As already mentioned, one needs a formal definition of the skeleton being used, since there are so many of them out there. This is the curve skeleton, see the aforementioned survey on 3D skeletons. Just saying one uses a '1D skeleton' is notu sufficient, there are other types of skeletons of 3D shapes which are one-dimensional in some sense or another (e.g. centerlines, Reeb graphs), and which are not of use in the current context. This can be added to the "Choice of skeleton" discussion.

We have revised “1D skeleton” into “curve skeleton”.

"Choise" -> "Choice"

Revised.

The choice of the skeletonization method of Au et al [36] is a good one. The method is also quite robust, pretty much parameter-free, directly accepts mesh models as inputs (unlike other methods that only work on voxels), and reasonably fast. I think the paper would be made stronger if these details were added. However, condition (1) is not extremely clearly justified: it is true that [36] can, in general, capture small-scale details as skeleton branches, but it doesn't always do so. Thus, unless the authors have a more formal/strict measurement of how much of (and/or what kind of) details they need to capture, the caveat that [36] is not guaranteed to \_always\_ capture a given type of detail, should be mentioned.

More discussion on the use of various types of skeletons is provided in the section for “Choice of skeletons” (Section 3), as well as regarding the particular choice of [36]. It is challenging to strictly measure how much details the skeleton needs to capture, as it amounts to solve another exponential-time searching problem, however, in practice, as shown in fig 4 and all the other examples in the paper, as long as the skeleton captures the topology and the general shape of the original model (i.e., does not degenerate), our method works well on these inputs. We have added a discussion of this in the Conclusion.

The beginning of Sec. 3 again refers to Fig. 4. I start thinking this is Fig. 3... And if so, can you sketch the overhand angle in that figure, so we know what to look for?

This notion is illustrated in Figure 4 of the revision.

I don't think it makes sense to refer to Tab. 2 at this point, as it contains many things which are described only later in the paper.

We have removed it.

Terminology: the paper refers here and there to skeleton arcs, skeleton pieces, and skeleton subtrees. I believe several of these notions overlap. In any case, please define them all clearly and upfront. Also in Fig. 3 or related text, define what is an overhang angle and overhang length.

We have changed “skeleton piece” into “skeleton arc” and clarified “arc” as “line segment” throughout. We have not used the term “subtree” anywhere. We have soothed the paper to make sure that the notions are not abused.

An illustration of “overhang angle” and “overhang length” has been added into Fig. 4 of the revision.

Problem statement: is the curve skeleton really decomposed into subgraphs, or are those parts actually subTREES? I think the distinction is important.

Subgraphs. Subtree is just a special case of the subgraph. We have added more explanation on this in the “Problem Statement” section.

"the least printable parts" -> the least number of printable parts, maybe? This repeats itself several times.

We have revised it throughout the paper.

Theorem 1: before one can understand this, please define the free variable(s). What is 'n' in this context? I see it's defined only implicitly and later on when one gives the proof of Theorem 1..

In the paragraph below “Problem statement”, we have defined ‘n’ as follows: “consider a Laplacian skeleton that is a fork with *n* arcs…”. To further clarify this, we have rephrased the statement of Theorem 1.

"Clique Cover ≤ p Skeleton Partition" Don't here some big-O symbols miss? I guess one can compare complexities, not methods themselves.. Apart from that, Theorem 1 would enormously benefit from a more detailed sketch, showing all the notations in the text in e.g. Fig. 4. I found it very hard to follow the text, and I wonder if I mis-guessed what some of the notations and expressions mean..

This is correct. We have checked the proof carefully. The expression and notations strictly follow the standard NP-hard proof.

Related to all above: I still don't quite get what is 'n' -- is it the number of so-called sample points along the curve skeleton, or is it the number of junctions of the skeleton? If it's the latter, then sure, an NP-complexity would be bad. If it's the former, I wonder if the practical complexity is really that high (given that for typical shapes one has say 10..15 junction points). Anyways, an explicit indication of what 'n' is in practice would make the case for the need of a faster method much stronger.

See above. We have rephrased the statement of Theorem 1 to clarify this.

"This completes the proof" -> fullstop missing.

We have added it back.

Sec. 4

I found the justification of minimizing the number of cuts as compared to minimizing the total cut-length quite meaningful and well-argued for. However, at this point, the reader wonders whether another important factor would not be the length of a cut itself: I can imagine, for instance, that very short cuts (small L\_i) are also undesirable, since they would make the gluing quite complex (and the result quite fragile). Can you comment on this?

That’s right. We have added more discussion on this in the revision. Refer to the second paragraph of Section 4.

For the rest, the description of the optimization problem is nicely done.

Also, the stochastic method proposed to decompose the skeleton (and the corresponding shape) is interesting, and makes sense in a heuristic way. Though, some elements need extra explanation:

-"We apply a training-and-learning procedure for the first k (say 1000) runs". I'm not sure I understand what are these runs (runs of what?)

We have changed “runs” into “times”. Refer to the second last paragraph in the section of “Skeleton Partition”.

-Mesh Partition: I \_think\_ I understood all sub-steps of the algorithm, but I am not sure. As for earlier algorithmic steps, a figure that indicates all the cases/decisions would enormously help here..

We have added one more figure (Fig. 7 ion the revision) to cover the cases that were not illustrated before.

-shape vertex v incident to a skeleton point p: What is this exactly? Is v one of the so-called feature points of p (i.e., one of the points of the surface that is at minimal distance from p), or is it something else? Having a clear definition would help reasoning about v.

We cannot find this in this text. We think you might want to distinguish the relationship between a node v and a point p on the mesh: During the Laplacian smoothing process, a set of points (usually form a cylindrical shape whose central axis lies on a skeleton) including p on the mesh may collapse into a node v. Refer to paragraph 2-3 of the section for “Mesh Partition”, we have also added more explanation on this.

-there are related methods to cutting a shape around a skeleton point, so that the cut meets various desired properties, such as smoothness and tightness (the latter being basically a minimal-length property). See e.g. [35], where geodesic cuts are used (many more papers use similar geodesic cuts for shape decomposition, see the 3D skeletons survey paper mentioned earlier). It would be thus good to comment, briefly, in which way the properties of the cuts proposed in the current paper are different (or similar) to the geodesic cuts, given that the latter are often used. If several such properties for the planar cuts proposed here are different from those of the geodesic cuts, this strengthens the motivation for having a new cut type. Also, a quite related cut model for shapes, based on their skeleton, is described in

Curve skeleton extraction from incomplete point cloud

(A. Tagliasacchi, H. Zhang, D. Cohen-Or), Proc. ACM SIGGRAPH, 2009

We have added the reference. Refer to Ref. [9].

As for the geodesic cuts, please briefly comment on the similarities/differences with the cuts proposed here, so we see why a new cut type is needed.

We use a planar cut instead of geodesic cuts which are often used to segment a mesh part since planar cuts are simple and much easier to glue when 3D printed. Refer to the first paragraph of the section for “mesh Partition”

-Fig. 8: please make the shape's surface less transparent, it's almost impossible to visualize it in the PDF.

We have revised it such that it is less transparent. Refer to Fig. 9 in the revision.

-Fig. 9: please indicate 'b' explicitly in the drawing by a line or cut, right now it's not clear what 'b' refers to geometrically... Also, drawing the normals n() in the figure would help a lot. Also in "n(c) points downwards", I guess c should be c\_3 (correct?)

We have revised the figure according to the suggestions. Now it is Fig. 10 in the revision.

Sec. 5

The results presented in this section are very rich and quite convincing. In particular, it is clearly illustrated how much (both qualitatively and quantitatively) the method saves in terms of support material. The case made by the metrics in Table 1 is quite clear. While the savings are not enormous (about a third on average in support material and time), they are also relatively speaking enough to justify the added value of the method.

Table 2: please indicate the unit used to measure the running time (from the text, it's unclear if it's minutes or something else). More importantly, please explain clearly in the text what is the training-and-learning process being used - it has been referred to earlier, but it's still unclear what is trained on, and on what are the learning results extrapolated (same model? family of models? etc).

Regarding the unit of running time, it is “second”, as indicated by “time(s)”. To make this clearer, we have put “s” aside each number.

We realize that this is not a strict training-and-learning process since it emphasizes too much on the “learning” part. To avoid any ambiguity we have rephrased this saying into “choosing the arcs by learning history record” in the revision. Refer to the second last paragraph of the section of “Skeleton Partition”.

For the practical evaluation, I miss a single thing: running times for the method used on a larger mesh. Modern meshes used in 3D printing can easily have hundreds of thousands or millions of vertices -- and the experiments shown here run on a mesh which is at most 35K vertices, which is quite small. It is really important to see how the method scales, in absolute running time terms, on a larger mesh - if it doesn't, one has to really reconsider the added-value of the proposed stochastic optimization.

The running time of the algorithm depends on the number of iterations, the number of mesh vertices and the number of skeleton arcs, the topology of the skeletons, the seed nodes of the subgraph used in each iteration, and the positions of the vertices that induce mesh partition. Refer to the third paragraph in the section of “Result”

Sec. 6: I think the pro's and con's are fairly treated. Still, the paper should highlight two extra aspects:

-a fair discussion of the practical running times for large models is needed, please see earlier remarks in this sense;

-a brief comment on the suitability of using other 3D curve-skeletonization methods is also in place. I assume that there's nothing in the Laplacian skeletonization method that makes it uniquely suited to this application. That is, other curve-skeletonization methods could be used too provided they're simple to use, fast, and capture details well. Please comment on this, as it clarifies the replicability of the method.

The discussion of the practical running times for large models is given in the third paragraph in the section of “Result”.

Yes. Laplacian skeleton is not the unique curve skeleton suited for describing the topology of the models, any curve skeleton that captures the geometric features of the model can be used as well. We have put this comment in the fourth item of the “Limitations and Future Work” section.

**Reviewer: 3**

Recommendation: Author Should Prepare A Major Revision For A Second Review

Comments:

While the analysis of the problem and the proposed method are interesting, the

manuscript presents several points that require, form my point of view, large changes.

First, I believe a small discussion about why printing thin shell is important. It could be to provide some functionality (in this case the cuts might be problematic). Or on the contrary, if the purpose of the thin shell is to limit the volume of material used, then the literature about interior carving for 3d printing should be discussed as it provide the same kind of benefits (see additional related works below).

Shell models are widely used in many mechanical and artistic lightweight designs where prototypes are needed. If a shell satisfies the need of the designer for prototype purpose, there is no need to fabricate a solid one with more materials. Some more discussion on this has been added into the revision. Refer to the third paragraph of the section for “Introduction”.

Then, the related work section ignore several recent works related to segmentation with additive manufacturing objectives. While several of them target packing, others highly reduce the need for support.

[1] M. Attene : Shapes In a Box: Disassembling 3D Objects for Efficient Packing and Fabrication, Computer Graphics forum, Vol. 34-8 (2015)

[2] M. Yao, Z. Chen, L. Luo, R. Wang, H. Wang : Level-Set-Based Partitioning and Packing Optimization of a Printable Model, ACM Transactions on Graphics, Vol 34-6 (2015)

[3] P. Song, B. Deng, Z. Wang, Z. Dong, W. Li, C-W Fu, L. Liu : CofiFab: Coarse-to-fine Fabrication of Large 3D Objects, ACM Trans. Graph., Vol 35-4 (2016)

[4] W-M Wang, C Zanni, L Kobbelt: Improved surface quality in 3d printing by optimizing the printing direction, Computer Graphics Forum, Vol 35-2 (2016)

Among those works, [2,4] could also be cited when talking about seams optimization.

Given the argument provided to justify the need to print without supports, the literature on

reducing the amount of material used to print the interior of an object should also be cited,

for instance :

[5] Wang, W., Wang, T.Y., Yang, Z., Liu, L., Tong, X., Tong, W., Deng, J., Chen, F., Liu, X.: Cost-effective printing of 3d objects with skin-frame structures. ACM Trans. Graph. (TOG) 32(6), 177 (2013)

[6] S. Hornus , Lefebvre S., Dumas J., Claux F.: Tight printable enclosures for additive manufacturing. Inria report, 2015.

[7] Y. Xie, X. Chen, Support-free interior carving for 3D printing, Visual Informatics, Vol 1-1 (2017)

All these references have been added and discussed properly in the section of “Related Work”.

While the overall explanations are clear enough, I find the explanation of the technical parts difficult to follow when it comes to the details that should be understood properly if one want to implement the method. First having a small paragraph that explain the main notations used in the section 4 might be really helpful. Besides, the same notion is used with different names, for instance rays, directed arcs and vectors are used to designate the direction of an edge of the skeleton. The relationship between the normal of the cutting plane and the printing direction (there are equals from what I understood) could be defined clearly at the beginning of section 4 as it is an important hypothesis for the algorithm that follow. This might also simplify some of the notation later on.

We do try to put a short paragraph in the beginning of section 4, we find that some notations can be summarized in the beginning of section 4, but many critical notations need to be defined with the aid of figures, and that it is better to define them when used. We have gone through the section to make sure that all notations are defined and used properly.

We have replaced “rays” and “vectors” by “directed arcs” throughout the paper. We have clarified the relationship between the normal of the cutting plane and the printing direction in the beginning of Section 4.

The two parts that are completely unclear for me are the one on the disc enclosure (column 1 page 6) and the choice of the cutting plane. For the second the problematic points are:

- "By feasible we require that a cutting plane does not cut through any other subgraph except for Hi." I do not understand this constraint and the explanation that follows does not really help me.

This is to avoid the scenario that a cutting plane cuts through the mesh component of another subgraph. However, in practice this may not be feasible if no cut can avoid cutting into another subgraph and its corresponding mesh component. In this case, a relaxation on this constraint is allowed. Refer to paragraph 6 of the section for “Mesh Partition”.

- I do not understand the choice of the cut when the intersection between the two cones is empty, nor do I see the use of a new notation F to describe the valid planes. Moreover, the need of the two cuts should be better explained.

Refer to paragraph 3-5 of the section for “Mesh Partition”, when the intersection is empty, we need to use two cuts to separate the mesh components correspond to *Hi* and *Hj*. Fig. 7 has been added to illustrate the valid planes.

The results section would require additional work. First, the benefits of the ‘training and learning’ step are not demonstrated. It would be a good idea to provide timing of the preprocessing step (in table 2) as most model do not come with a skeleton. The material reduction provided in Table 1 do not provide a fair comparison with state of the art as there exist more ‘material saving’ supports structures that the one used for the comparisons and methods to limit interior material used. In Table 1, the time save of the last line is wrong as the segmented object take more time to be printed. Finally, better pictures of the printed results (Fig12) should be provided.

Regarding the problem with Table 1, we just realized that the data of the original models (the second and third column) has not been changed: at the very beginning, all models were tested directly based on their given orientation without minimizing their support volumes. The material use and print time of the original models belong to this case. In the revision, we have provided the correct data of the original models when their printing orientations are provided by the Meshmixer software.

In order to provide better pictures, we refabricated all the partitioned models and assembled the components in a more elegant manner. Refer to Fig. 13 for this.

One of the most problematic aspect is that some important points are not discussed in the limitations. First the end of a skeleton branches (vertex of the skeleton of valence 1) are not handled properly when it comes to supports, indeed the surface of the model around such points is not tubular (spherical cap for instance) - work done in [6] could provide a solution for this case if the 'cavity' inside of the object do not have a functional purpose. This is clearly seen in example such as the octopus or the gargoyle head. It should also be stated that the method assumes constant radius along skeletons to provide guarantee on the absence of supports.

Indeed, the method works well for models with constant radius along skeletons. We put this as a limitation in the item (vi) of the section for “Limitations and Future Work”.

Finally, the fact that the final segmentation can always be assembled is not discussed. Some more details about the assembly process could also be provided: does an assembly order always exist and some word on the difficulty to match correctly different parts (which is even more difficult when the cut is planar - a part of previous work on segmentation for 3d printing introduce connectors to limit this problem).

Allowing male and female connectors on the interfaces may induce more support structures as these facets are used as the base. With the planar cuts, small tricks made on the interfaces of a pair of parts can lead to an efficient matching, e.g., marking each interface by extruding a small spike into the void region of the shell model; and gluing two parts at their interfaces by matching the spikes. We have put this observation in first paragraph of the section for “Limitations and Future Work”.

Finally, the manuscript would require some additional proofreading,

both for english, for instance :

choice between a/an (example p3 l14)

further => furthermore

else otherwise

p1 l49 : pyramidical => pyramidal

p2 l17 : expendential ?

p2 l42 : among which geometric features captur[es/ing] shape concaveness are mostly exploited

p3 l27 : forces that [/are] inherent to …

p3 l26 : by an angle [of no large/not larger] than theta

p3 l30 : into a proper number of support-free subgraphs [leads/leading] to a partition of the model …

p2 l26 : models whose shapes ?merit? well-defined skeletons

=> models whose shapes can be well-described by skeletons ?

and for other small errors that are disturbing, such as:

p4 l59 : Given ?three? integer numbers c, d, let …

Fig 9 : The [green=>colored?] lines …

All are revised.

Overall, the proposed approach is interesting but the manuscript requires several improvements : related works, better explanation, additional information in the result section, more discussions on the limitation.

We have made a large amount of changes to the paper according to the comments.